FP7-PEOPLE-2012-IAPP

mesh-wise

Self-organising MESH Networking with Heterogeneous Wireless Access

D1.1: Mathematical models for capacity characterisation of wireless mesh networks

Contractual date of delivery to REA: Month 12
Actual date of delivery to REA: March 26th 2014
Lead beneficiary: LiU
Nature: PUBLIC
Version: 1.0

Project Name: Self-organising MESH Networking with Heterogeneous Wireless Access
Acronym: MESH-WISE

Start date of project: 01/03/2013 **Duration:** 48 Months
Project no.: 324515

Project funded by the
European Union under the
People: Marie Curie Industry-Academia
Partnerships and Pathways (IAPP)
Programme of the 7th Framework
FP7-PEOPLE-2012-IAPP

MARIE CURIE ACTIONS
SEVENTH FRAMEWORK PROGRAMME

Document Properties

Document ID	PEOPLE-2012-IAPP-324515-MESH-WISE-D1.1
Document Title	*Project Management Plan*
Lead Beneficiary	Linköping University (LiU)
Editor(s)	Di Yuan - LiU
Work Package No.	1
Work Package Title	Fundamental performance characterization
Nature	Report
Number of Pages	32
Dissemination Level	PU
Revision History	Version 1
Contributors	LiU: Vangelis Angelakis, Di Yuan ULUND: Björn Landfeldt, Yuan Li, Michal Pioro FORTH: N/A. Forthnet: Manolis Stratakis MobiMESH: Antonio Capone, Stefano Napoli,

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0 Executive summary

Wireless mesh networking forms a promising solution for future broadband access. To design scalable and high-capacity solutions, much research and development (R&D) efforts remain. This report is devoted to summarizing results of applying mathematical optimization for fundamental capacity characterization of wireless mesh networks. The report focuses on mathematical optimization models for three optimization aspects, and a case study to illustrate the structure of optimal solutions.

The first optimization aspect is resource allocation, which is vital for achieving high throughput in wireless mesh networks. We present mixed integer programming models for link transmission scheduling, rate allocation, routing design, with extensions to directional antennas and channel assignment. The models are formulated based on compatible sets, to allow for a decomposition of the problem into assigning time slots to compatible sets and generating compatible sets. Utilizing this property of the model, we apply the column generation method as the main solution approach.

Then, we consider the fairness aspect for multiple flows in wireless mesh networks. We take the max-min fairness criterion as the objective, which not only helps to improve the fairness but also enables to deal with traffic uncertainty. We provide a mixed integer programming model for fair resource allocation and propose an algorithm which solves the model sequentially to achieve max-min fairness.

Finally, we provide mixed integer programming models for capacity characterization of metric-based routing. In this problem, link metrics used for route selection are not preset, but subject to optimization. We consider the average network delay as the objective and characterize open shortest path first (OSPF) type of routing for wireless mesh networks.

The presented models covers a wide range of capacity optimization tasks. Solving the optimization models is a significant step for benchmarking and performance evaluation in designing wireless mesh networks.

1 Introduction

Wireless Mesh Networks (WMNs) offer a cost-efficient solution for providing Internet access in metropolitan and residential areas. Designing WMNs is attaching a significant amount of research and development efforts from the physical layer to the application layer [38].

Many existing wireless technologies have been or will be extended to support the paradigm of WMN. The IEEE 802.11 WLAN is one of the most popular technology to implement WMN in municipal wireless networks, broadband home networking, enterprise networking, etc [44]. Mesh architectures are specifically considered in IEEE 802.11s, and in IEEE 802.16 Wireless Metropolitan Area Networks to construct the backbone in a cost-efficient way [15]. WMN is also considered a promising solution for the backhaul of next generation cellular systems Long Term Evolution (LTE) [47]. Moreover, some short range radio technologies, e.g., IEEE 802.15.4 also adopts mesh-type topologies [34].

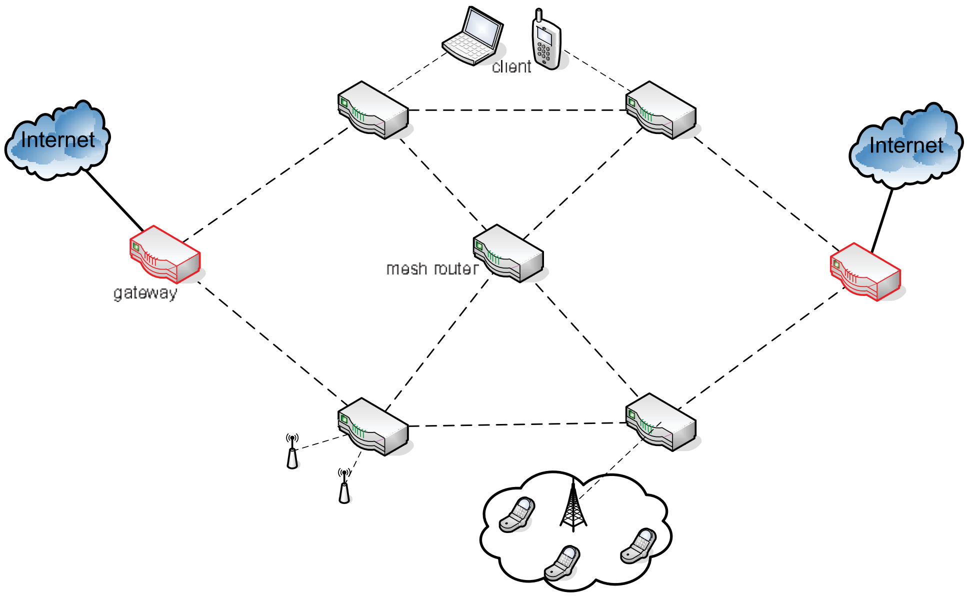

Figure 1: A typical WMN architecture.

There are three main devices in WMNs: gateways, mesh routers and mesh clients, which are connected by wireless links. The backbone of a WMN is composed of mesh routers and gateways. Gateways are typically connected to the Internet directly and mesh routers are connected to gateways, either directly or through other mesh routers. Mesh clients (i.e., end users) are distributed in the network area. They are connected to gateways, or mesh routers and gain access to the Internet via the paths connecting the routers to the gateways. A typical architecture of a WMN is shown in Figure 1. The key features of WMNs are summarized below [3].

- Multi-hop networks. The data from a mesh client can pass multiple mesh routers to arrive at another mesh client that is far away. Thus, due to the multi-hop nature, WMNs can have large coverage area to provide access in rural areas.

- Lost cost, self-healing and self-organization. WMNs can make use of the off-the-shelf wireless communication equipments to construct the network without investment on new types of equipments. Due to its flexible architecture, WMN can be easily deployed and reconfigured. The meshed topology also makes the network resilient and fault tolerate.

- Integration with other networks. Utilizing the bridge functionality of mesh routers, WMNs can be integrated with existing network technologies, e.g., WiFi, WiMAX, ZigBee, cellular systems, etc., forming heterogeneous networks.

WMNs share some characteristics of ad-hoc networks, since both of them are multi-hop networks and have no wired infrastructure. However, there are many differences between them. In wireless ad-hoc networks, each node is not only a user which can produce traffic but also a router capable of routing and configuration. For WMNs, the traffic is produced by mesh clients and routed by mesh routers. Another key difference is that the nodes in wireless ad-hoc networks are highly mobile, and thus the topology is changed with the movement of users. This also poses challenges to the design of routing protocols. In WMNs, mesh routers are stationary in most cases, and thus the topology is stable. Although mesh clients are moving, they do not affect the network topology. Therefore, design solutions for wireless ad-hoc networks cannot be directly ported to WMNs. Moreover, traffic management in WMNs has to deal with, to a large extent, flows between mesh routers and gateways.

One role of WMNs is to serve as wireless backbone with appropriate quality of service. Therefore, traffic optimization and related resource allocation are important engineering tasks. In this report, mathematical programming is applied to study resource optimization in WMNs, in terms of representing a range of optimization problems by efficient mixed-integer programming or linear programming formulations for capacity characterization and computation. Solving the optimization models enables to obtain insights into optimal design of WMNs and give performance bounds which can be used as guidelines for network engineering.

For system characterization, we take into account a number of important concepts in radio networks. These include access schemes based on time division, power control, link rate adaptation, and the channel assignment [35]. Most multiple access techniques for mesh-type wireless networks can be classified into two categories: CSMA (carrier Sense Multiple Access) and TDMA (Time Division Multiple Access). From the throughput standpoint, CSMA is inferior to TDMA [48]. In CSMA, each sender should sense the channel before transmitting and it can transmit only when the channel is idle, with the issues of the hidden terminal problem and exposed terminal problem. These issues cause delay and degrades bandwidth. For TDMA, each node can successfully transmit in the assigned time slots. By proper design of the time division scheme, TDMA can achieve much better spatial reuse and thus higher throughput. Because the target of the study is on the characterization of the potentially achievable capacity of WMNs, optimization for TDMA-based multiple access is considered in this report.

Link rate adaptation is a widely used technology in radio networks [7]. This capability allows each transmitter to adapt its modulation and coding schemes and thereby change the data rate with respect to the channel condition. By adjusting link rates, the network throughput is improved.

The power control mechanism means that each network node can tune its transmitting power [42]. Transmitting power has high impact on network performance since the a high transmitting power of a node enables more reliable transmission and high data rate for the node, but causes much stronger interference to other nodes. There is hence a trade-off between the intensity of the power and the interference, which can be resolved by optimization in power control.

The use of multiple orthogonal channels is a direct way of scaling up network capacity [23]. Having multiple orthogonal channels available adds another dimension to performance optimization, in terms of assigning the channels to the transmissions such as the spectrum resource reuse is optimal for throughput and capacity.

Routing is a classic optimization task in communication networks. For WMNs, it amounts to determining the route(s) from each WMN route to a gateway, such that the traffic on the routes can be resource-efficiently supported by the underlying wireless links.

In this report, mathematical, mixed integer programming models are developed for a number of resource allocation problems in TDMA-based WMNs: link scheduling, routing, rate adaption, as well as their combinations. The models, that are comprehensive and as well as fundamental for capacity characterization, represent a cross-layer view of WMN resource optimization.

For given traffic demands of WMN clients, the bandwidth requirement can be translated into the traffic that must be transmitted per frame. The objective is thus the minimal length of the frame (i.e., the number of time slots) to accommodate the required demands. In one time slot, a set of links are scheduled to transmit in parallel. We introduce a notion, referred to as compatible set, to represent any set of links that can transmit successfully and concurrently for their respective chosen powers and rates. We use this notation to formulate the problem with a non-compact model which can be solved by column generation (that is, to generate compatible sets). Channel assignment is then taken into account to enable more compatible sets to be active in the same time slot.

To address fairness, the max-min flow problem in WMNs is studied. We consider flows between gateways and mesh routers. The objective is to achieve, lexicographically, the maximum of a bandwidth allocation vector, in which an element is the bandwidth assigned to a flow. With this fairness objective, mixed-integer linear programming models for the corresponding resource allocation problem is derived. The notion of compatible set is still present in the optimization model. Given a time duration, solving the model gives the optimal time partition among the compatible sets. The column generation method is used to solve the optimization problem. This procedure is then embedded into an algorithm that achieves max-min fairness.

Finally, we present optimization models for metric-based path selection and routing in WMNs. In this problem, the routing has to be optimized together with link metrics. That is, link metrics, used for computing shortest path routes, are not preset but have to be optimized.

This report is organized as follows. Basic notations and interference models are given in Section 2. Next, Section 3 introduces optimization models based on compatible sets for resource allocation in TDMA-based WMNs. The max-min flow fairness problem is studied in 3 and the joint optimization of routing and link metrics is presented in Section 5. An illustrative case study is given in Section 6. Finally, Section 7 concludes this report.

2 System Model

In this report, the topology of a WMN is modeled as a bi-directional graph $(\mathcal{V}, \mathcal{E})$. $\mathcal{V}$ is the set of nodes, composed of a set of gateways $\mathcal{G}$ and a set of mesh routers $\mathcal{R}$, i.e., $\mathcal{V} = \mathcal{G} \cup \mathcal{R}$. The nodes are connected by the directed (radio) links $\mathcal{E}$. The head and tail node of a link e are represented as $a(e)$ and $b(e)$ respectively. The graph is bi-directional, i.e., if link $(a(e), b(e))$ exist in the graph, link $(b(e), a(e))$ also exists in the graph. Further, the set of links incident to node v is denoted as $\delta(v)$, including the set of outgoing links $\delta^+(v)$ and the set of incoming links $\delta^-(v)$, i.e., $\delta(v) = \delta^+(v) \cup \delta^-(v)$.

For any active link, the date rate depends on the selected *modulation and coding schemes* (MCSs). Table 1 shows a set of available MCSs of IEEE 802.11a [16]. Let $\mathcal{M}$ denote the set of MCSs. As shown in the table, each MCS $m \in \mathcal{M}$ corresponds to a unique date rate s^m and a SINR threshold $\widehat{\gamma}^m$. The SINR is expressed in dB scale which can be converted to linear scale by $\gamma^m = 10^{\frac{\widehat{\gamma}^m}{10}}$.

A fundamental aspect in studying wireless networks is how to model interference. There has been a large amount of work studying various interference models (e.g., [18, 19]). A summary is provided below.

Table 1: Modulation and coding schemes in IEEE 802.11a.

m	MCS	raw bitrate s^m	SINR threshold $\widehat{\gamma}^m$
0	BPSK 1/2	6 Mbps	3.5 dB
1	BPSK 3/4	9 Mbps	6.5 dB
2	QPSK 1/2	12 Mbps	6.6 dB
3	QPSK 3/4	18 Mbps	9.5 dB
4	16-QAM 1/2	24 Mbps	12.8 dB
5	16-QAM 3/4	36 Mbps	16.2 dB
6	64-QAM 2/3	48 Mbps	20.3 dB
7	64-QAM 3/4	54 Mbps	22.1 dB

- *K-hop interference model:*
  Two links cannot active simultaneously when they are within K hops. For example, the 1-hop case often refers to the node-exclusive interference model, i.e., a node cannot transmit and receive on two separate links concurrently.

- *Protocol interference model:*
  A node can successfully receive signals from a transmitter if the node is in the transmission range of the transmitter and no other nodes are active in its interference range. Take Figure 2 as a example, link (1,2) and link (5,6) can not be active at the same time since node 2 is in the interference range of node 5. However, link (3,4) and link (7,8) can be active with link (1,2) at the same time.

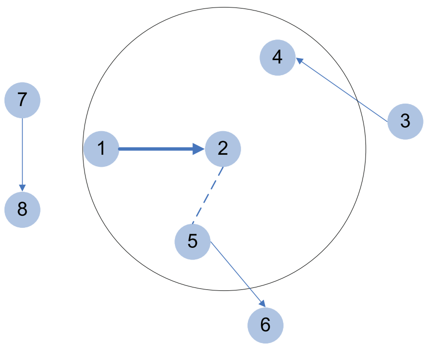

Figure 2: An example illustrating the protocol interference model.

- *Physical interference model:*
  For a signal transmitted from node i to node j, whether or not node j can successfully decode the signal depends on the relation between the power of this signal and the sum of all interference. This is expressed by the *signal-to-interference-plus-noise ratio* (SINR) constraint (1), where $\mathcal{A}$ is a set of simultaneously active links, and P_{ij} is the power received at node j when node i is transmitting. Each link in $\mathcal{A}$ should satisfy the SINR constraint. That is, the ratio of the power received at $b(e)$ from $a(e)$ to the noise power η plus the interference observed at $b(e)$ from other transmitting nodes must be greater than or equal to the SINR threshold γ^m.

$$\frac{P_{a(e)b(e)}}{\eta + \sum_{v \in \mathcal{V} \setminus \{a(e)\}} P_{vb(e)}} \geq \gamma^m, \; e \in \mathcal{A} \tag{1}$$

To show its difference with the protocol interference model, we can look at Figure (3) in which the interference from nodes 3, 5, and 7 are considered at node 2. The SINR constraint is then checked by equation (1) to determine whether the link (1,2) can be active or not.

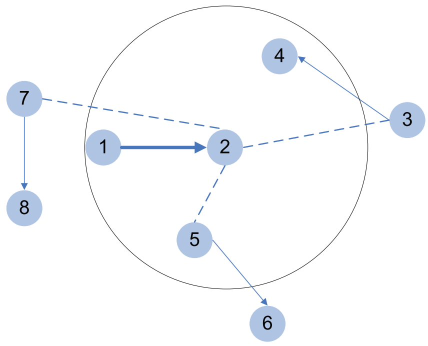

Figure 3: An example illustrating the physical interference model.

The K-hop interference model can be used in some wireless networks. For example the 1-hop and 2-hop interference models are applicable to FH-CDMA and IEEE 802.11 DSSS based networks, respectively. However, this model estimates the interference relationship based on hops, which is far from exact to character the interference. For the protocol model, the interference relationship of links can be effectively represented by a graph, called the *conflict graph*. In a conflict graph, a node represents a link in WMN. A link in conflict graph means that two nodes (two links in WMN) cannot active at the same time. The conflict graph can be directly applied to the protocol interference model and the K-hop interference model since whether or not two links can be active simultaneously in WMN only depends on the geometrical relation between the two links. For the physical interference model, however, whether or not a link can be active depends on all other active links not one of them.

Besides, the protocol model and the K-hop model are not exact interference model as accumulated interference from other active nodes are not taken into account. The physical model can exactly character this kind of interference and thus is the model we adopt in this report. We will incorporate the SINR constraint (1) in the optimization models.

We end this section by summarizing some basic notations in Table 2. The notation will be used throughout the report.

Table 2: A summary of basic notation.

$\mathcal{V}, \mathcal{E}, \mathcal{G}, \mathcal{R}$	sets of nodes, links, gateways and mesh routers, respectively
$a(e), b(e)$	head and tail nodes of link $e \in \mathcal{E}$, respectively
$\delta(v), \delta^+(v), \delta^-(v)$	set of links incident to, outgoing from, and incoming to node $v \in \mathcal{V}$, respectively
P_{ij}	power received at node j from node i
η	thermal noise power
$\mathcal{M}$	set of available MCSs, $m \in \mathcal{M}$
γ^m	SINR threshold for MCS $m \in \mathcal{M}$
s^m	data rate of MCS $m \in \mathcal{M}$
D_r	the volume of traffic for mesh router $r \in \mathcal{R}$
$\mathcal{Q}_r$	the set of paths for the flow to mesh router r
$o(r)$	the origination (a gateway) of the demand for mesh router $r \in \mathcal{R}$
$\mathcal{I}$	the list of compatible sets
$\mathcal{C}_i$	a set of simultaneously active links which is defined as a compatible set
B_{ei}	the data rate of link e in compatible set i
$\mathcal{J}$	the set of orthogonal channels and $\lvert J \rvert$ is the number of available orthogonal channels
K	the number of interfaces in each node

3 Optimization Models for Resource Allocation

In this section we present the resource allocation problem in wireless mesh networks when the demand flows are given for each mesh router. For each mesh router $r \in \mathcal{R}$, there is a demand with volume D_r, originating from some gateway $o(r) \in \mathcal{G}$ and terminates at the mesh router r.

We present optimization models for joint scheduling, power control, rate adaptation, routing, as well as channel assignment. We firstly introduce the optimization model for transmission scheduling, and then extend the model to routing, rate adaptation and power control. Channel assignment can be added on top of the solution produced from the models.

The variables used in this section are summarizes in Table 3.

3.1 Link Scheduling

Link scheduling is a fundamental topic in wireless networks. The problem complexity of link scheduling under various interference model is analyzed in [45, 49]. A survey of link scheduling algorithms in WMNs is given in [17]. In this section, the problem is approached from the viewpoint of mathematical programming. We consider minimizing the number of time slots required to accommodate all demand flows by optimizing transmission scheduling. Do so is coherent with the task of maximizing capacity. The routing paths for each mesh router is assumed to be given. In the basic formulation, the transmitting powers of all nodes are uniform and constant, and only one MCS is available, i.e., there is one available data rate, denoted by s, and its SINR threshold denoted by γ.

The transmission schedule is devised based on the notion of compatible set, which is a set of simultaneously active links without violating the SINR constraint (1). Let $\mathcal{I}$ be a list of compatible sets. The date rate of link $e \in \mathcal{E}$ in compatible set $i \in \mathcal{I}$ is denoted as B_{ei}. If link e belongs to the compatible set $\mathcal{C}_i, i \in \mathcal{I}$, $B_{ei} = s$, where s is the given data rate. If link e does not belong to $\mathcal{C}_i$, we set $B_{ei} = 0$. We introduce constant $\Delta_{er}, r \in \mathcal{R}, e \in \mathcal{E}$ to represent whether or not the demand

Table 3: Variables used in Section 3.

t_i	integer variable, representing the number of time slots in which compatible set $\mathcal{C}(i)$ is active
Y_e	binary variable, representing whether or not link e is active
X_v	binary variable, representing whether or not if node v is active
z_{er}	continuous variable, representing the amount of flow for mesh router r passing through link e
Z_{rq}	continuous variable, representing the amount of flow for mesh router r passing through path q
y_e^m	binary variable, representing whether or not link e is active and uses MCS m
U_e^m	binary variable, representing whether or not link e uses MCS m
p_v	continuous variable, representing the transmitting power of node v
w_{it}^j	binary variable, representing whether or not compatible set $\mathcal{C}(i)$ is assigned to slot t with channel j
u_{it}	binary variable, representing whether or not compatible set $\mathcal{C}(i)$ is assigned to slot t
λ_t	binary variable, representing whether or not time slot t is used
κ_{vj}	binary variable, representing whether or not node v uses channel j

flow for mesh router r passes through link e. The basic formulation for link scheduling (LS) is given in (2). In the formulation, $t_i, i \in \mathcal{I}$, is an integer variable, representing the number of time slots in which compatible set $\mathcal{C}(i)$ is active.

$$\textbf{LS} \qquad \text{minimize } \sum_{i\in\mathcal{I}} t_i \tag{2a}$$

$$\text{subject to: } [\alpha_e] \sum_{r\in\mathcal{R}} D_r \Delta_{er} \le \sum_{i\in\mathcal{I}} B_{ei} t_i, \; e \in \mathcal{E} \tag{2b}$$

The objective of model LS is to minimize the number of time slots used by all compatible sets. The entities in the brackets are not part of the constraint but denote, for convenience, the dual variables for the linear relaxation of model LS. Constraint (2b) assures that the amount of traffic on link e should not exceed its capacity. The capacity of a link e is defined as the sum of data rates of compatible sets, in which link e is active, times the corresponding number of time slots.

An optimal solution of LS consists of finding a list of compatible sets $\mathcal{C}_i^*, i \in \mathcal{I}$ and the corresponding usage of time slots t_i^*. The formulation (2) is non-compact since the number of compatible sets grows exponentially with the size of the network. Thus, in general, the list of all compatible sets cannot be predefined in approaching the optimum. However, appropriate compatible sets can be generated by column generation during branch-and-bound (B&B) process while solving the linear relaxations of model LS. This combination of B&B and column generation is called branch-and-price (B&P). An example of applying B&P in transmission scheduling is provided in MESH-WISE project publication [32], and the application of column generation in wireless networks can be found in MESH-WISE project publication [51].

The B&P algorithm will be time-consuming for solving the model when the network instances are large. A practical solution can be obtained by solving the linear relaxation of model LS. In the linear relaxation, the integer variables $t_i, i \in \mathcal{I}$ are relaxed to be continuous and the model becomes a linear programming formulation. Then one can use the column generation method to generate compatible sets until the linear relaxation problem is optimally solved. If in the obtained solution some $t_i, i \in \mathcal{I}$ are fractional, rounding can be applied to convert them into their nearest integers, to obtain a feasible integer solution. Below details of using the column generation method are provided.

For the linear relaxation of model (2), the dual problem is derived and shown in (3).

$$\text{maximize } \sum_{e\in\mathcal{E}}\sum_{r\in\mathcal{R}} D_r\Delta_{er}\alpha_e \tag{3a}$$
$$\text{subject to: } \sum_{e\in\mathcal{E}} \alpha_e B_{ei} \le 1,\ i \in \mathcal{I} \tag{3b}$$
$$\alpha_e \ge 0, \quad e \in \mathcal{E} \tag{3c}$$

By (3), the number of constraints (3b) depends on the number of compatible sets. Suppose α^* is an optimal solution of the dual problem for a given list of compatible sets $\mathcal{I}$, the compatible set generation process consists of finding a compatible set i outside the current list such that constraint (3b) is violated, i.e., $\sum_{e\in\mathcal{E}} \alpha_e B_{ei} > 1$. Then adding such a compatible set to the problem can potentially increase the optimal dual objective, and thereby improve the optimal primal objective value because of strong duality in linear programming [25].

The way of finding such a compatible set is shown in formulation (4), denoted by CSG (Compatible Set Generation). The objective is to maximize the left-hand side of constraint (3b) for the current optimal dual variables α^*. If the obtained objective of model (4) is greater than 1, then the found compatible set is added to the linear relaxation of (2), and (2) is then optimized again. The compatible set generation procedure stops until no compatible set violating (3b) exists.

In the formulation (4), the variables are defined as follows.

$$Y_e = \begin{cases} 1, & \text{if link } e \text{ is active} \\ 0, & \text{otherwise} \end{cases} \qquad X_v = \begin{cases} 1, & \text{if node } v \text{ is active} \\ 0, & \text{otherwise} \end{cases}$$

CSG

$$\text{maximize } s\sum_{e\in\mathcal{E}} \alpha^*_e Y_e \tag{4a}$$
$$\text{subject to: } \sum_{e\in\delta(v)} Y_e \le 1, \quad v \in \mathcal{V} \tag{4b}$$
$$X_v = \sum_{e\in\delta^+(v)} Y_e, \quad v \in \mathcal{V} \tag{4c}$$
$$\eta + \sum_{v\in\mathcal{V}\setminus\{a(e)\}} P_{vb(e)}X_v \le \frac{1}{\gamma}P_{a(e)b(e)}Y_e + (1-Y_e)M_e, \quad e \in \mathcal{E} \tag{4d}$$

The constraints have the following effects.
(4b): among all links incident to node v, only one of them can be active.
(4c): node v is active if one of outgoing links is active.
(4d): this is the SINR constraint. The constant $M_e = \eta + \sum_{v\in\mathcal{E}\setminus\{a(e)\}} P_{vb(e)}$. If link e is active ($Y_e = 1$), then the SINR condition must be satisfied. In case $Y_e = 0$, the inequality holds true trivially.

Model CSG needs to be solved many times to in order to reach the optimum of the linear relaxation of model LS. Thus it makes sense to find ways to accelerate computing CSG. In fact CSG can be strengthened by introducing two families of inequalities: cover-type inequality and matching inequality.

Consider constraint (4d) in CSG. If a link e is active, then (4d) can be transformed to a *knapsack constraint*: $\sum_{v\in\mathcal{V}\setminus\{a(e)\}} P_{vb(e)}X_v \le \frac{1}{\gamma}P_{a(e)b(e)} - \eta$. The knapsack constraint can be reformulated using cover-type inequality. A set $C \subseteq \mathcal{V}\setminus\{a(e)\}$ is called a cover if $\sum_{v\in C} P_{vb(e)}X_v > \frac{1}{\gamma}P_{a(e)b(e)} - \eta$. The SINR constraint requires that at most $|C|-1$ nodes in $|C|$ can be active. The SINR cover inequality is expressed as $\sum_{v\in C} X_v \le |C| - Y_e$. Note that the inclusion of Y_e in the right-hand side restricts the activation of nodes in C to at most $|C|-1$, only if link e itself is active.

Model CSG extended with cover inequality is shown in the formulation (5).

CSG-C

$$\text{maximize } s\sum_{e\in\mathcal{E}}\alpha_e^* Y_e \tag{5a}$$

$$\text{subject to: } \sum_{e\in\delta(v)} Y_e \leq 1, \quad v\in\mathcal{V} \tag{5b}$$

$$\sum_{e\in\delta^+(v)} Y_e = X_v, \quad v\in\mathcal{V} \tag{5c}$$

$$\sum_{v\in C} X_v \leq |C| - Y_e$$

$$e\in\mathcal{E}, C\subseteq\mathcal{V}: \sum_{v\in C} P_{vb(e)} > \frac{P_{a(e)b(e)}}{\gamma} - \eta \tag{5d}$$

This model can be solved by constraint generation, i.e., iteratively generating cover which violates constraint (5d), and adding the generated cover to the model and solving the model again. This process stops until no violated cover can be generated. The cover inequality can be further enhanced by the lifted cover inequality. More details for generating cover inequality and lifted cover inequality can be found in [10].

To enhance CSG with matching equality, one need to construct an undirected graph $(\mathcal{V},\mathcal{L})$ from the original bi-directional graph $(\mathcal{V},\mathcal{E})$. The set of nodes remains. Each edge $l\in\mathcal{L}$ corresponds to a pair of bi-directional links of $\mathcal{E}$, where two oppositely directed links $f'(l), f''(l)$ corresponds to undirected edge $l\in\mathcal{L}$. Model CSG extended with matching inequality is formulated in (6).

In model (6), $\mathcal{O}(v)$ represents the family of all subsets of $\mathcal{V}$ with odd cardinality greater than 1. $U\in\mathcal{O}(v)$ is an odd-set and $E[U]$ denotes the set of edges in $\mathcal{L}$ with both end nodes in U. Inequality (6c) is called the matching inequality.

CSG-M

$$\text{maximize } s\sum_{e\in\mathcal{E}}\alpha_e^* Y_e \tag{6a}$$

$$\text{subject to: } \sum_{l\in\delta(v)} x_l \leq 1, \quad v\in\mathcal{V} \tag{6b}$$

$$\sum_{l\in E[U]} x_l \leq \frac{|U|-1}{2}, \quad U\in\mathcal{O}(\mathcal{V}) \tag{6c}$$

$$Y_{s'(l)} + Y_{s''(l)} \leq x_l, \quad l\in\mathcal{L} \tag{6d}$$

$$\sum_{e\in\delta^+(v)} Y_e = X_v, \quad v\in\mathcal{V} \tag{6e}$$

$$\eta + \sum_{v\in\mathcal{V}\setminus\{a(e)\}} P_{vb(e)}X_v \leq \frac{1}{\gamma}P_{a(e)b(e)}Y_e + (1-Y_e)M_e, \quad e\in\mathcal{E} \tag{6f}$$

Again, this model can be solved by constraint generation, i.e., by iteratively generating an odd-set violating constraint (6c). More details for generating violated odd-set can be found in [31].

3.2 Extension to Routing, Rate Adaptation, Power Control, and Directional Antenna

We extend model LS (2) for optimizing routing, that is, the routing path is not preset for the mesh routers. For each demand flow of a mesh router $r\in\mathcal{R}$, only the origin $o(r)$ and the destination r are given. Introducing nonnegative continuous variable z_{er} to represent the amount of flow for mesh router r passing through link e, the model for joint link scheduling and routing (LSR) is formulated in (7).

$$\textbf{LSR} \quad \text{minimize } \sum_{i\in\mathcal{I}} t_i \tag{7a}$$

$$\text{subject to: } \sum_{r\in\mathcal{R}} z_{er} \le \sum_{i\in\mathcal{I}} B_{ei} t_i,\ e\in\mathcal{E} \tag{7b}$$

$$\sum_{e\in\delta^+(v)} z_{er} - \sum_{e\in\delta^-(v)} z_{er} = \begin{cases} D_r, & v=o(r) \\ -D_r, & v=r\ , r\in\mathcal{R} \\ 0, & \text{otherwise} \end{cases} \tag{7c}$$

The objective in LSR is the same as the objective of LS, i.e., to minimize the number of time slots used by activated compatible sets. Constraint (7b) assures that the load of link e does not exceed its capacity. Constraint (7c) is the flow conservation rule. For each demand flow of mesh router $r \in \mathcal{R}$, the amount of traffic outgoing from a node minus the incoming traffic equals to the required traffic at an origin node, zero at any intermediate node, and the negative value of the demand at a destination. By solving model (7), we obtain optimal routing paths for the traffic of each mesh router $r \in \mathcal{R}$. Note that there may be several routing paths used for the demand of a mesh router.

Formulation (7) uses link flow variables z_{er}, of which the number is $|\mathcal{E}| \times |\mathcal{R}|$. The number of such variables increases fast with respect to network size. Thus it is time-consuming to solve the model with a mixed-integer programming solver. As an alternative, (7) can be reformulated using path variables. Let $\mathcal{Q}_r$ be the set of all possible paths for the demand of mesh router $r \in \mathcal{R}$. We define continuous variables Z_{rq} to represent the amount of flow for mesh router r on path $q \in \mathcal{Q}_r$. Constant $\nabla_{eq}, e \in \mathcal{E}, q \in \mathcal{Q}_r$ represents whether or not link e is included in path q. The formulation with path variables for joint scheduling and routing is presented in (8).

$$\textbf{LSRP} \quad \text{minimize } \sum_{i\in\mathcal{I}} t_i \tag{8a}$$

$$\text{subject to: } \sum_{r\in\mathcal{R}} \sum_{q\in\mathcal{Q}_r} Z_{rq}\nabla_{eq} \le \sum_{i\in\mathcal{I}} B_{ei} t_i,\ e\in\mathcal{E} \tag{8b}$$

$$\sum_{q\in\mathcal{Q}_r} Z_{rq} = D_r,\ r\in\mathcal{R} \tag{8c}$$

In model (8), constraint (8b) ensures that the amount of flows passing through a link e should not exceed its capacity. Constraint (8b) assures that all demands are realized.

The number of paths in $\mathcal{Q}$ will increase exponentially in network size. To solve the model, we can generate paths in the similar way as generating compatible sets, by deriving the dual problem of the linear program version of (8). It turns out the subproblem for generating paths amounts to solving a shortest path problem for each demand, with link weights being the optimal dual multipliers associated with constraint (8b). More details of shortest path generation are given in [39].

Model LS can also be extended to incorporate the link rate assignment. The link rate assignment means that each node can select an appropriate modulation and coding scheme (MCS) for its transmission. There are two basic ways of MCS assignment: static MCS assignment and dynamic MCS assignment. In the static case, each link uses a fixed MCS whenever it is active. For dynamic MCS assignment, also known as rate adaptation, each link can adjust its MCS slot-by-slot [4,5,37].

Recall that the available set of MCS is denoted by $\mathcal{M}$, and each MCS $m \in \mathcal{M}$ corresponds to a unique data rate s^m and an SINR threshold γ^m. Given a list of compatible sets, if link e belongs to compatible set $\mathcal{C}(i)$ and uses MCS m, then the data rate of this link is $B_{ei} = B^m$; otherwise $B_{ei} = 0$.

To incorporate MCS assignment in the model, we extend the model of generating compatible set, i.e.,CSG. We introduce binary variables y_e^m to represent whether link e is active and uses MCS

$m \in \mathcal{M}$ ($y_e^m = 1$) or not ($y_e^m = 0$). Compatible set generation with rate adaptation (CSG/RA) is formulated in (9).

$$
\begin{aligned}
\textbf{CSG/RA} \quad & \text{maximize } \textstyle\sum_{e\in\mathcal{E}} \alpha_e^* \sum_{m\in\mathcal{M}} B^m y_e^m && \text{(9a)}\\
& \text{subject to: } Y_e = \textstyle\sum_{m\in\mathcal{M}} y_e^m, \quad e \in \mathcal{E} && \text{(9b)}\\
& \textstyle\sum_{e\in\delta(v)} Y_e \le 1, \quad v \in \mathcal{V} && \text{(9c)}\\
& X_v = \textstyle\sum_{e\in\delta^+(v)} Y_e, \quad v \in \mathcal{V} && \text{(9d)}\\
& \eta + \textstyle\sum_{v\in\mathcal{V}\setminus\{a(e)\}} P_{vb(e)} X_v \\
& \qquad \le \frac{1}{\gamma^m} P_{a(e)b(e)} y_e^m + (1 - y_e^m) M_{em}, \quad e \in \mathcal{E}, m \in \mathcal{M} && \text{(9e)}
\end{aligned}
$$

In the objective function of CSG/RA, the data rate of link e is computed as $\sum_{m\in\mathcal{M}} B^m y_e^m$. Compared with model CSG, a new constraint (9b) is added to assure that a link can only use one MCS. The SINR constraint is also changed to (9e), in which the SINR threshold for link e depends on the MCS used by it. Variable y_e^m is used to replace Y_e in the right-hand side. The value of M_{em} is set to $\eta + \sum_{v\in\mathcal{E}\setminus\{a(e)\}} P_{vb(e)}$.

Model CSG/RA can be used to generate compatible set for either LS or LSR if the rate adaptation is adopted.

Although link rate adaptation through dynamic use of MCS is potentially a mechanism to react to changing conditions on a radio channel, fixed assignment of MCSs is of high significance. First, fixed MCS assignment can perform better than the adaptive one for some cases since the latter depends on how well the channel condition can be detected or estimated. Also, hardware complexity of routers decreases for fixed MCS assignment. Moreover, at the network design stage, adaptive MCS may not be applicable as only average channel condition is considered, and solutions derived from dynamic MCS with assumptions on channel may not be practical.

To formulate scheduling with static MCS assignment, binary variables U_e^m are introduced to represent whether link e uses MCS m ($U_e^m = 1$) or not ($U_e^m = 0$). We make modifications in model LS (2) and obtain formulation (10). Constraint (10c) assures that each link e can only use one MCS and constraint (10d) makes sure that a link will use the MCS assigned to it ($U_e^m = 1$) for all compatible set where the link is active.

$$
\begin{aligned}
\textbf{SRA} \quad & \text{minimize } \textstyle\sum_{i\in\mathcal{I}} t_i && \text{(10a)}\\
& \text{subject to: } \textstyle\sum_{r\in\mathcal{R}} D_r \Delta_{er} \le \sum_{i\in\mathcal{I}} B_{ei} t_i, \ e \in \mathcal{E} && \text{(10b)}\\
& \textstyle\sum_{m\in\mathcal{M}} U_e^m = 1, \ e \in \mathcal{E} && \text{(10c)}\\
& \textstyle\sum_{m\in\mathcal{M}} r_{ei}^m t_i \le U_e^m T, \ e \in \mathcal{E}, m \in \mathcal{M} && \text{(10d)}
\end{aligned}
$$

In SRA, except for integer variables t_i, there are also binary variables U. Similar to the basic scheduling formulation, the model can be solved by B&P. The pricing problem corresponding to this problem is given in formulation CSG/RA, see (9).

When power control is available, the transmitting power for each node becomes a variable, see, for example, MESH-WISE publication [33]. Let $p_v, v \in \mathcal{V}$ be the continuous variables representing the transmitting powers of the nodes. The maximum transmission power available is denoted by P^{max}. Power parameter $P_{a(e)b(e)}$ will now be replaced by $p_{a(e)} G_{a(e)b(e)}$ where the constant $G_{a(e)b(e)}$ is the channel gain between node $a(e)$ and $b(e)$.

To incorporate power control into the problem, one need to reformulate the subproblem for generating the compatible set, CSG. The new formulation for generating compatible set with power control (CSG/PC) is shown in formulation (11).

$$\textbf{CSG/PC} \quad \text{maximize } s\sum_{e\in\mathcal{E}} \alpha_e^* Y_e \tag{11a}$$
subject to: (4b), (4c)
$$p_v \leq P^{max} X_v, \quad v \in \mathcal{V} \tag{11b}$$
$$\eta + \sum_{v\in\mathcal{V}\setminus\{a(e)\}} p_v G_{vb(e)} \leq \frac{1}{\gamma} p_{a(e)} G_{a(e)b(e)} + (1-Y_e)M_e, \quad e \in \mathcal{E} \tag{11c}$$

Compared with model CSG, constraint (11b) is added to make sure that the transmitting power does not exceed the maximum value P^{max}. It also assures that when node v is inactive, the transmitting power is equal to 0. The SINR constraint is reformulated in constraint (11c) in which $p_v, v \in \mathcal{V}$ are variables and $M_e = \eta + \sum_{v\in\mathcal{V}\setminus\{a(e)\}} P^{max} G_{vb(e)}$.

Directional antenna is a technology that has the ability to improve network capacity [50]. A directional antenna can focus the transmission energy between a communicating pair of nodes, resulting in increased spatial reuse and transmission range in comparison to omni-directional antennas. Hence the transmitter can limit the interference it generates at the non-intended receivers, and, similarly, a receiver experiences reduced the interference received by non-intended transmitters. In the following we extend the optimization model by taking into account directional antennas.

The gain of the directional antenna is characterized by the antenna pattern, which reflects the relative power in any direction compared with omni-directional antennas. There are two basic terms related to antenna pattern: the *peak gain* which is the maximum gain over all directions, and the beam width which is the angle between the two directions where a half peak gain is obtained. We employ a typical directional antenna pattern which is generated by the sin-function (12) [29].

$$\bar{G}(\theta) = \frac{\sin(N(\frac{\pi}{2}\cos\theta + \frac{1}{2}\varphi)}{\sin(\frac{\pi}{4}\cos\theta + \frac{1}{2}\varphi)} \tag{12}$$

In (12), N is the number of elements in the antenna array, θ measures the angle drifts off the direction of peak gain for a direction and $\bar{G}(\theta)$ is the antenna gain in this direction. The values of φ controls the direction of the main beam. The peak gain is obtained when $\varphi = -\frac{\pi}{2}\cos\theta$ and the value is then equal to N. By adjusting N, we achieve antenna patterns with different beam widths and peak gains. An example antenna pattern is given in Figure 4, with $\varphi = -\frac{\pi}{2}$ and $N = 5$. The beam width is 120° and the peak gain is 5. Note that the antenna gain for omni-directional antenna equals one.

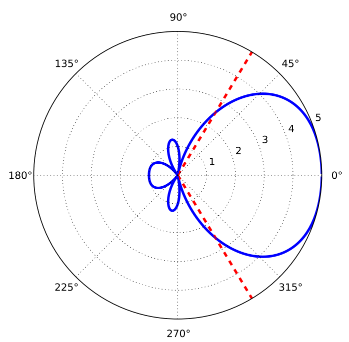

Figure 4: An example of directional antenna radiation pattern.

The topology of WMN is given and each node knows its position, as well as that of all the neighbors. When directional antennas are used in WMNs, it is reasonable to assume that the transmitter can point its main beam to its intended receiver. The delay for turning the directional antenna becomes negligible due to the advanced antenna technologies, e.g., switch antennas or adaptive antennas. Since the antenna radiation pattern is not uniform, the interference arriving at a node depends on the directions of the directional antennas deployed by the non-intended transmitters. Consider two active links as in Figure 5, the interference received at $b(e)$ from $a(e')$ is computed as $\bar{P}_{ae'}b(e) = P_{ae'}b(e)\bar{G}(\theta_{a(e')b(e')b(e)}\theta_{b(e)a(e)a(e')})$. Note that the directional antennas on $a(e)$ and $b(e)$ point to each other and the same holds for $a(e')$ and $b(e')$. For WMNs, the value of $\bar{G}$ for any pair of links can be pre-computed.

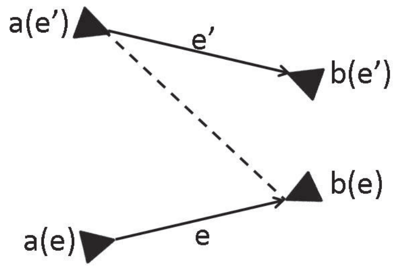

Figure 5: communication with directional antennas

For considering directional antennas in WMNs, we reformulate the SINR constraint in model CSG for generating compatible sets. The new model CSG/DA is formulated in (13). One can observe that the only difference between model CSG/DA and model CSG is the way to calculate the power.

CSG/DA maximize $s\sum_{e\in\mathcal{E}}\alpha_e^* Y_e$ (13a)

subject to: $\sum_{e\in\delta(v)} Y_e \le 1, \quad v\in\mathcal{V}$ (13b)

$$X_v = \sum_{e\in\delta^+(v)} Y_e, \quad v\in\mathcal{V} \tag{13c}$$

$$\eta + \sum_{v\in\mathcal{V}\setminus\{a(e)\}} \bar{P}_{vb(e)}X_v \le \frac{1}{\gamma}\bar{P}_{a(e)b(e)}Y_e + (1-Y_e)M_e, \quad e\in\mathcal{E} \tag{13d}$$

Previous results of joint routing and scheduling with directional antennas can be found in [11]. Details of optimization for effective placement of directional antennas are given in MESH-WISE project publication [30].

3.3 Channel Assignment

The solutions generated by the models in Section 3.1 and Section 3.2 are based on the assumption of a single channel. The benefits of multiple orthogonal channels are not exploited. When multiple channels are available, communications on different channels can exist concurrently without interfering each other. For WMNs, the use of multiple radio interfaces and multiple channels can greatly increase the network capacity. Previous works in the literatures on the effects of multiple interfaces on network capacity can be found in [14, 24, 27, 43, 46].

For multi-radio multi-channel WMNs, two interfaces can communicate only when they are tuned to the same channel. Given a limited number of interfaces and channels, the allocation among them has to be optimized for improving network performance.

For link scheduling, the previous models generate are single-channel frames in which one activated compatible set is assigned to every time slot in order to satisfy traffic demands. In this section, the study addresses constructing a multi-channel frame by assigning channels to active compatible sets and then compatible sets to slots. The objective remains to minimize the total number of required slots. Note that the SINR constraint is taken into account in the construction of compatible sets.

There are two basic approaches for channel assignment: dynamic assignment and static assignment. For dynamic channel assignment, the interface can use different channels in different slots, while in the static case the interface uses one channel for the entire frame duration. We present optimization models for both both cases. Let $\mathcal{J}$ be the set of orthogonal channels and $|J|$ the number of available orthogonal channels. Let K be number of interfaces in each node. The list of compatible sets $\mathcal{I}$ can be obtained by solving the previous models. We use parameter $\beta_{iv}, i \in \mathcal{I}, v \in \mathcal{V}$ to denote whether or not node v is transmitting in compatible set $\mathcal{C}(i)$, with $\delta^+(v) \cap C(i) \neq \emptyset$. The set of multi-channel time slots is denoted as $\mathcal{T}$.

The formulation for dynamic channel assignment (DCA) is given in (14). In this model, we introduce binary variables $u_{it}, i \in \mathcal{I}, t \in \mathcal{T}$, to represent whether the compatible set $\mathcal{C}(i)$ is assignment to slot t ($u_{it} = 1$) or not ($u_{it} = 0$). Other binary variables are $\lambda_t, t \in \mathcal{T}$, representing whether slot t is used ($\lambda_t = 1$) or not ($\lambda_t = 0$).

$$
\textbf{DCA} \qquad
\begin{aligned}
\text{minimize } & \textstyle\sum_{t\in\mathcal{T}} \lambda_t && (14a)\\
\text{subject to: } & \textstyle\sum_{t\in\mathcal{T}} u_{it} = 1, \quad i \in \mathcal{I} && (14b)\\
& \textstyle\sum_{i\in\mathcal{I}} u_{it} \le |\mathcal{J}|, \quad t \in \mathcal{T} && (14c)\\
& \textstyle\sum_{i\in\mathcal{I}} \beta_{iv} u_{it} \le K, \quad v \in \mathcal{V}, t \in \mathcal{T} && (14d)\\
& u_{it} \le \lambda_t, \quad i \in \mathcal{I}, t \in \mathcal{T} && (14e)
\end{aligned}
$$

The constraints are derived for the following effects.
(14b): each compatible set is assigned a slot.
(14c): the number of used orthogonal channels is no greater than H.
(14d): the number of used interfaces for each node is no greater than K.
(14e): a slot is consumed if any compatible set is active in the slot.

The static channel assignment (SCA) problem is formulated in (15). The difference between SCA and DCA is that in the former a node cannot change channel from one slot to another. In model SCA, we introduce binary variables $w_{it}^j, j \in \mathcal{J}, i \in \mathcal{I}, t \in \mathcal{T}$ to represent whether the compatible set $\mathcal{C}(i)$ is assignment to slot t with channel j ($w_{it}^j = 1$) or not ($w_{it}^j = 0$). Binary variables $\kappa_{vj}, v \in \mathcal{V}, j \in \mathcal{J}$ are introduced to represent whether node v uses channel j ($\kappa_{vj} = 1$) or not ($\kappa_{vj} = 0$).

$$
\textbf{SCA} \qquad
\begin{aligned}
\text{minimize } & \textstyle\sum_{t\in\mathcal{T}} \lambda_t && (15a)\\
\text{subject to: } & \textstyle\sum_{j\in\mathcal{J}} \sum_{t\in\mathcal{T}} w_{it}^j = 1, \quad i \in \mathcal{I} && (15b)\\
& \textstyle\sum_{i\in\mathcal{I}} w_{it}^j \le 1, \quad j \in \mathcal{J}, t \in \mathcal{T} && (15c)\\
& \textstyle\sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} w_{it}^j \le |\mathcal{J}|, \quad t \in \mathcal{T} && (15d)\\
& \textstyle\sum_{t\in\mathcal{T}} \beta_{iv} w_{it}^j \le \kappa_{vj}, \quad j \in \mathcal{J}, v \in \mathcal{V}, i \in \mathcal{I} && (15e)\\
& \textstyle\sum_{j\in\mathcal{J}} \kappa_{vj} \le K, \quad v \in \mathcal{V} && (15f)\\
& w_{it}^j \le \lambda_t, \quad j \in \mathcal{J}, i \in \mathcal{I}, t \in \mathcal{T} && (15g)
\end{aligned}
$$

Constraints (15b), (15d), (15f), and (15g) have their counterparts in model DCA. The new constraint (15c) states that in each multi-channel time slot, at most one compatible set can be assigned with a channel. Constraint (15e) assures that each node v can only use one channel over all time slots. As solving the models exactly has been challenging, suboptimal solutions can be found by heuristics [8].

4 Optimization Models for Max-Min Flow Fairness

Fairness is an important performance metric in WMNs [2, 26, 30]. WMN optimization driven by solely maximizing the throughput objective often causes servere unfairness, i.e., users near to gateways obtain much more capacity than users far away from gateways whereas users paying the same charge for the WMN access expect the same quality of service no matter the location. Therefore, there is a trade-off between capacity and fairness. In this study, the max-min fairness (MMF) for throughput optimization is applied. For the general introduction of MMF, we refer to [13, 28]. A MMF solution means that no route can gain higher flow without having to decrease the flow on a route for which the current flow is smaller. In WMNs, the traffic mainly refers to the flow between mesh routers and mesh gateways. We consider throughput as a vector of bandwidths assigned to the downstream of the traffic demands from a mesh gateway to mesh routers. The demands for traffic are assumed to be elastic, and hence can consume any bandwidth assigned. The throughput optimization with MMF can be translated to finding a lexicographically maximal flow allocation vector. Using this as the objective address well the fairness aspect as well as the the throughput of each mesh router.

Figure 6 uses an example to illustrate the MMF solution. In the figure, node 1 is the gateway, and nodes 2 and 3 are mesh routers. Flows are denoted by f_2 and f_3 for mesh routers 2 and 3, respectively. Link $(1, 2)$ and link $(2, 3)$ cannot be active simultaneously. Given three time slots and data rate 6 Mbps, the optimal solution corresponding to maximizing capacity is that $f_2 = 18$ Mbps, $f_3 = 0$ Mbps. The optimal MMF solution, in contrast, assigns $f_2 = f_3 = 6$ Mbps.

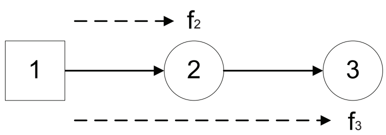

Figure 6: An illustration of MMF.

Another benefit for applying MMF in WMNs is that the demands do not have to be assumed to be given, as were assumed in the models models presented in Section 3. For unknown or uncertain traffic, some optimization methods, e.g. robust optimization and stochastic optimization [12], are proposed. However, these methods are not well suited for the design of WMNs, because the profile of the traffic is sometimes hard to model, as mesh clients connected to mesh routers tend to have a large diversity. Applying MMF in assigning bandwidth to WMN routes is a good way to deal with traffic uncertainty. The traffic in WMN can then be regarded elastic, and represents demand aggregation of mesh clients attached to mesh routers [41].

For the purpose of modeling, we need to define the link capacity which can be characterized by link scheduling, with optional use of power control and rate adaptation. The overall task is to achieve MMF with resource allocation. We make use of the notion of compatible set in the model. Let $\mathcal{Q}$ be the set of routes. The amount of traffic for each route is unknown and we define variables f_q to denote the amount of flow for route q. The routing path for each mesh router is assumed to be given given without loss of generality, since the model can be easily extended for optimizing routing. With the assumption, the constants $\Delta_{eq}, e \in \mathcal{E}, q \in \mathcal{Q}$ represent whether or not the route q passes through link e. Instead of using individual time slot, the entire time horizon of of network operation (or the length of a frame) T is given. Continuous variables $z_i, i \in \mathcal{I}$ are used to define the portion of time T used by compatible set $\mathcal{C}(i)$. The MMF flow optimization problem is formulated in (16).

$$\textbf{MMF} \quad \text{lexicographically maximize } \langle f \rangle = (\langle f \rangle_1, \langle f \rangle_2, ..., \langle f \rangle_{|\mathcal{Q}|}) \tag{16a}$$

$$\text{subject to: } \textstyle\sum_{i \in \mathcal{I}} z_i = T \tag{16b}$$

$$[\alpha_e] \textstyle\sum_{q \in \mathcal{Q}} f_q \Delta_{eq} \leq \sum_{i \in \mathcal{I}} B_{ei} z_i, \quad e \in \mathcal{E} \tag{16c}$$

In (16a), $\langle f \rangle = (\langle f \rangle_1, \langle f \rangle_2, ..., \langle f \rangle_{|\mathcal{Q}|})$ represents the vector $f = (f_1, f_2, ..., f_{|\mathcal{Q}|})$ sorted in non-decreasing order. The objective is the lexicographical maximization of the sorted vector, with $\langle f \rangle_1 \leq \langle f \rangle_2 \leq \cdots \leq \langle f \rangle_{|\mathcal{Q}|}$. We say that a vector sorted in non-decreasing order f' is lexicographically greater than another vector f'' sorted in non-decreasing order, if there is an index k $(1 \leq k \leq |\mathcal{Q}|)$ such that $f'_i = f''_i, i = 1, 2, ..., k-1$ and $f'_k \geq f''_k$. For example, a sorted vector $(1, 5, 10)$ is lexicographically greater than sorted vector $(1, 2, 100)$.

Model (16) is solved sequentially using the *conditional means approach*, see [36]. The main idea is to maximize the entries of $\langle f \rangle$ one by one. When the current entry is to be optimized, the previous optimized entries are kept and their optimized values. The algorithm solving the model is detailed in Algorithm 1 below.

Algorithm 1 Algorithm for solving model (16)
Step 0: Set $\mathcal{B} = \emptyset$ (blocking demand)
Step 1: Solve the mathematical program:

$$\text{maximize } F \tag{17a}$$

$$\text{subject to: } [\rho_q] \; F \leq f_q, \quad q \in \mathcal{Q} \setminus \mathcal{B} \tag{17b}$$

$$f_q = f^*_q, \quad q \in \mathcal{B} \tag{17c}$$

$$\textstyle\sum_{q \in \mathcal{Q}} f_q \Delta_{eq} \leq \sum_{i \in \mathcal{I}} B_{ei} z_i, \quad e \in \mathcal{E} \tag{17d}$$

Step 2: Denote the resulting optimal objective of model (17) by F^* and $\rho^*_q, q \in \mathcal{Q}$. If $\rho^*_q > 0$ and $q \in \mathcal{Q} \setminus \mathcal{B}, \mathcal{B} = \mathcal{B} \cup q$ and $f^*_q = F^*$.
Step 3: If $\mathcal{B} \neq \mathcal{Q}$, go back to **Step 1**. Otherwise, stop and the optimal MMF solution is given by $f^* = (f^*_1, f^*_2, ..., f^*_{|\mathcal{Q}|})$

Note that in Algorithm 1, model (17) is a non-compact linear programming formulation. It can be solved by column generation, i.e., compatible set generation. Model CSG and all its extensions introduced in Section (3) can be used for the purpose of generating compatible sets; the details of this process has been presented in Section (3). We refer to MESH-WISE project publications [30–32].

5 Variale Metric Routing Design

For high-capacity WMN, it is of high significance that the routing paths are optimized [6, 40]. For given traffic demands, the routing determines the links that transmissions take place, and the amount of traffic on each link, which in turn affects the interference. This report has derived optimization models for joint scheduling and routing, see LSR and LSRP in Section 3. The links are of uniform preference in these models and the flow can be split on several paths. In LSRP, it is possible to incorporate path preference, e.g., routing could be restricted to paths with minimum number of hops to the destination. However, the hop count is not an efficient routing metric in wireless mesh networks since many characteristics of the wireless medium are not taken into account, such as interference and channel condition. Moreover, using hop count as the routing metric can result in some bottleneck links that cause large delay. In this section we consider to optimize routing together with setting link preferences, also known as link metrics.

Although WMNs inherits many characteristics of wireless ad-hoc networks, WMNs has fair stable network architecture, and carry relative stable backbone-like traffic. These aspects should be taken into account for routing protocol design. The reactive routing strategy used in wireless ad-hoc networks is not suitable for WMNs. In reactive routing, the route discovery message is sent when there is a demand arises between nodes. Such kind of flooding messages for on-demand route discovery brings large overhead in case of WMNs. Another limitation is use of hop count as the routing metric. A second type of routing used in wireless ad-hoc networks is proactive routing protocol, in which flooding messages are sent when failure of link occurs. The proactive routing protocol is a table-driven protocol for which each node maintains a routing table recording the next hop information for routes of all destinations. Many of the proactive routing protocols have been designed and implemented for WMNs, see [1, 21]. However, traditional proactive routing protocols still use hop count as the routing metric. In our work, we incorporate routing metric selection into the optimization of routing for minimizing the network delay.

In this section, joint optimization of routing path selection and link metric design based on the Open Shortest Path First (OSPF) routing protocol, one of the most commonly used protocol, see, for example [9, 20, 39]. Although OSPF is widely used in wired networks, it fits WMNs as well as its architecture is similar to the wired backbone networks due to the stable, decentralized, non-hierarchical structures.

Toward the end of routing together with link metric consideration, a rule used in the OSPF routing protocol, the equal-cost multi-path split rule (ECMP), is applied when multiple paths exhibit the same and minimum length (measured in the link metrics) to the destination are available. In ECMP, the demand for a source-destination pair will be split among these paths. Figure 7 provides an example to illustrate the ECMP rule. Without loss of generality, we assume that all link weights are equal to one, i.e., the special case of hop count as the link metric. For the demand from node 1 to node 7, there are three shortest paths between them: $1-2-5-7$, $1-3-5-7$, and $1-4-6-7$. According to the ECMP rule, each path will carry $\frac{1}{3}$ of the total traffic. However, if we consider the traffic from node 7 to node 1, the flow allocation among the paths has a different pattern. The two outing path $7-5-3-1$ and $7-5-2-1$ carry $\frac{1}{4}$ of the total traffic each. Route $7-6-4-1$ will carry $\frac{1}{2}$ of the total traffic. The reason is that the respective incoming flows is split equally at node 7 and node 5.

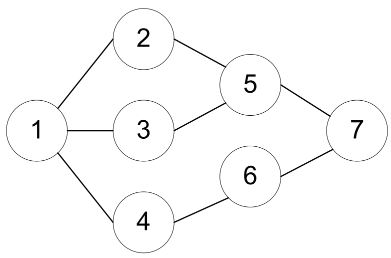

Figure 7: An illustration of equal-cost multi-path split.

For routing and link metric optimization, the topology, traffic demands, and link capacity are assumed to be given. Unlike the multi-commodity flow formulation presented in models LSR and LSRPG, flows in metric-driven routing depend on the link metric system which is also to be optimized. Let $f_r, r \in \mathcal{R}$ be the volume of demand for mesh router r, $\mathcal{Q}_r$ be the set of routes for demand of mesh router r, and parameters δ_{erq} be an indicator representing whether link e is in route q of demand for mesh router r.

We introduce the link metric vector $w = (w_1, w_2, ..., w_{|\mathcal{E}|})$, in which the entries are optimization variables. The link metric w should be positive to avoid loops in the resulting optimal paths. The path flow variables depend on the link metric vector and are represented as $x_{rq}(w), q \in \mathcal{Q}_r, r \in \mathcal{R}$.

Consider measuring traffic demand and the link capacity in packets per second (pps) which are integral-valued. If for each link the arrival of packets follows the Poisson distribution, the average packet delay on link e can be derived as $1/(c_e - y_e)$, in which y_e is the packet arrival rate. This is obtained according to the Little's law of the M/M/1 queuing model [22]. With independent link arrivals, the delay is considered on a link-by-link basis. Then, the average packet delay for the networks is expressed by equation (18).

$$F = \frac{1}{\sum_{r\in\mathcal{R}} f_r} \sum_{r\in\mathcal{R}} \sum_{q\in\mathcal{Q}} x_{rq}(w) \sum_{e\in\mathcal{E}} \delta_{erq} \frac{1}{c_e - y_e} = \frac{1}{\sum_{r\in\mathcal{R}} f_r} \sum_{e\in\mathcal{E}} \frac{y_e}{c_e - y_e} \tag{18}$$

In addition, there is bound on the delay of each link, e.g., the delay for each link is not longer than T seconds, with $T > 1/c_e$. Then, $y_e \le c_e - 1/T$. It can be rewritten as $y_e \le \gamma_e c_e$ with $\gamma_e = 1 - 1/(c_e T)$. Parameter γ_e is also know as the link utilization adjustment factor.

A straightforward formalization of routing optimization with link metric selection for delay minimization is given by (19).

$$\begin{aligned}
\text{minimize } & F = \textstyle\sum_{e\in\mathcal{E}} \frac{y_e}{c_e - y_e} && \text{(19a)}\\
\text{subject to: } & \textstyle\sum_{r\in\mathcal{R}} x_{rq}(w) = f_r, \quad q \in \mathcal{Q} && \text{(19b)}\\
& y_e = \textstyle\sum_{r\in\mathcal{R}} \sum_{q\in\mathcal{Q}_r} x_{rq}(w)\delta_{edq}, \quad e \in \mathcal{E} && \text{(19c)}\\
& y_e \le \gamma_e c_e, \quad e \in \mathcal{E} && \text{(19d)}
\end{aligned}$$

Problem described by (19) consists in finding link metrics which induce the flow allocation vector. The objective is obtained from equation (18) by simply omitting the constant $\frac{1}{\sum_{r\in\mathcal{R}} f_r}$. Equality (19b) assures that each demand is realized using metric-based paths. Equality (19c) defines the packet arrival rate of link e (i.e., link load) which is the sum of all demands passing through the link. Inequality (19d) expresses the bound of delay on each link.

Note that (19) is not a mathematical programming formulation, because of the implicit relation denoted by $x_{rp}(w)$. Furthermore, the objective function (19a) is non-linear. This function, $F_e(y_e) = y_e/(c_e - y_e)$, can be approximated by a piecewise link function, as shown in (20) [39].

$$f(y_e) = \begin{cases} y_e, \quad 0 \le y_e/c_e < \frac{1}{3} \\ 3y_e - \frac{2}{3}c_e, \quad \frac{1}{3} \le y_e/c_e < \frac{2}{3} \\ 10y_e - \frac{16}{3}c_e, \quad \frac{2}{3} \le y_e/c_e < \frac{9}{10} \\ 70y_e - \frac{178}{3}c_e, \quad \frac{9}{10} \le y_e/c_e < 1 \\ 500y_e - \frac{1468}{3}c_e, \quad 1 \le y_e/c_e < \frac{11}{10} \\ 5000y_e - \frac{16318}{3}c_e, \quad \frac{11}{10} \le y_e/c_e < \infty \end{cases} \tag{20}$$

Next, we introduce auxiliary variables r_e to represent $f(y_e)$. Formulation (19) can then be re-stated by (21).

$$
\begin{aligned}
\text{minimize } & F = \textstyle\sum_{e\in\mathcal{E}} r_e && \text{(21a)}\\
\text{subject to: } & \textstyle\sum_{r\in\mathcal{R}} x_{rq}(w) = f_r, \quad q \in \mathcal{Q} && \text{(21b)}\\
& y_e = \textstyle\sum_{r\in\mathcal{R}} \sum_{q\in\mathcal{Q}} x_{rq(w)}\delta_{erq}, \quad e \in \mathcal{E} && \text{(21c)}\\
& y_e \le \gamma_e c_e, \quad e \in \mathcal{E} && \text{(21d)}\\
& r_e \ge y_e, \quad e \in \mathcal{E} && \text{(21e)}\\
& r_e \ge 3y_e - \frac{2}{3}c_e, \quad e \in \mathcal{E} && \text{(21f)}\\
& r_e \ge 10y_e - \frac{16}{3}c_e, \quad e \in \mathcal{E} && \text{(21g)}\\
& r_e \ge 70y_e - \frac{178}{3}c_e, \quad e \in \mathcal{E} && \text{(21h)}\\
& r_e \ge 500y_e - \frac{1468}{3}c_e, \quad e \in \mathcal{E} && \text{(21i)}\\
& r_e \ge 5000y_e - \frac{16318}{3}c_e, \quad e \in \mathcal{E} && \text{(21j)}
\end{aligned}
$$

Still, model (21) cannot be directly solved due to variables $x_{rq}(w)$, for which the relation between link metrics and the resulting flows are not explicitly formulated. In order to arrive at an explicit formulation of (21), other variables will be used in place of $x_{rq}(w)$. We introduce parameter $D_{vr}, v \in \mathcal{V}, r \in \mathcal{R}$ to represent the demand flow from node v to node r. The new variables are defined as follows.

Variables

z_{er}	non-negative and continuous variable, representing the amount of demand to node r on link e
w_e	non-negative and continuous variable, representing the metric of link e
u_{vr}	non-negative and continuous variable, representing the the length of the shortest path from node v to node r
X_{vr}	non-negative and continuous variable, representing the amount of flow assigned to each shortest path from no
Y_{er}	binary variable, representing whether or not e is on a shortest path to node r

The explicit formulation for joint optimization of link metric and routing (LMR) is presented in (22).

$$
\begin{aligned}
\textbf{LMR} \quad \text{minimize } & F = \textstyle\sum_{e\in\mathcal{E}} r_e && \text{(22a)}\\
\text{subject to: } & \textstyle\sum_{e\in\delta^-(r)} z_{er} = \sum_{v\neq r} D_{vr}, \quad r \in \mathcal{R} && \text{(22b)}\\
& \textstyle\sum_{e\in\delta^+(v)} z_{er} - \sum_{e\in\delta^-(v)} z_{er} = D_{vr}, v \neq r, r \in \mathcal{R}, v \in \mathcal{V} && \text{(22c)}\\
& \textstyle\sum_{r\in\mathcal{R}} z_{er} \le c_e, \quad e \in \mathcal{E} && \text{(22d)}\\
& 0 \le X_{a(e)r} - z_{er} \le (1 - Y_{er}) \textstyle\sum_{v\in\mathcal{V}} D_{vr}, \quad r \in \mathcal{R}, e \in \mathcal{E} && \text{(22e)}\\
& z_{er} \le Y_{er} \textstyle\sum_{v\in\mathcal{V}} D_{vr}, \quad r \in \mathcal{R}, e \in \mathcal{E} && \text{(22f)}\\
& 0 \le u_{b(e)r} + w_e - u_{a(e)r} \le (1 - Y_{er})M, \quad r \in \mathcal{R}, e \in \mathcal{E} && \text{(22g)}\\
& 1 - Y_{er} \le u_{b(e)r} + w_e - u_{a(e)r}, \quad r \in \mathcal{R}, e \in \mathcal{E} && \text{(22h)}\\
& w_e \ge 1, \quad e \in \mathcal{E} && \text{(22i)}\\
& y_e = \textstyle\sum_{r\in\mathcal{R}} z_{er} && \text{(22j)}\\
& \text{(21e)} - \text{(21j)}
\end{aligned}
$$

In model (22), the objective is to minimize the average packet delay of the network. The constraints are defined for the following effects.

- (22b): The total amount of flow incoming to node r and for node r has to be equal to the total flow generated in all remaining nodes destined to node r.
- (22c): For the flows destined to node r, the amount of flow outgoing from node v minus the amount of flow incoming to node v must be equal to the required demand from node v to node r. Note that if node v is not a gateway, D_{vr} is zero. Thus the flow conservation is induced by this constraint.
- (22d): The load of a link does not exceed its capacity.
- (22e): If a link e belongs to a shortest path for node r, then the flow for node r on link e equals $X_{a(e)r}$ – a value common to all outgoing links (belonging to the shortest paths to node r) from node $a(e)$. The ECMP rule is implied by this constraint.
- (22f): If link e does not belong to a shortest path to node r, then there should be no demand for node r on link e.
- (22g): If link e belongs to a shortest path for node r ($Y_{er} = 1$), the metric of link e should be counted for the computation of the shortest path length for node r. The constant M is a large number to makes the inequality hold when $Y_{er} = 0$.
- (22h): If link e does not belong to a shortest path for node r ($Y_{er} = 0$), the metric of link e should not be counted for the computation of the shortest path for node r. In this case, the metric of link e should be greater than or equals to one which is assured by (22i).
- (22j): This equation expresses the load for each link.
- The other constraints (21e) – (21j) are inherited from model (21) for linearization of the network delay function.

Solving (22) obtains the optimal routing paths for all mesh routers and the link metrics, for characterizing the achievable capacity utilization with metric-driven routing. This optimal solution can be embedded into the OSPF-like routing protocols for WMNs.

6 A case study

In this section we illustrate some solutions obtained from the derived models. A mesh network example as shown in Figure 8, deployed within an area of $500\ m \times 500\ m$. The squares represent gateways and the circles represent mesh routers.

At the physical layer, the example WMN uses OFDM at the 5 GHz band. The available modulation and coding schemes (MCSs) are given in Table 1. The way to compute the pass loss is shown in equation (23), where p_{vw} (in dBm) is the power received at node w from node v, p_v (in dBm) is the transmitting power of node v, and d_{vw} (in km) is the distance between the two nodes.

$$p_{vw} = p_v - 140.046 - 40 \cdot \log d_{vw} \tag{23}$$

First, we illustrate joint optimization of link transmission scheduling and dynamic rate selection, i.e., the combination of model (2) and model (9). The routing is given for each mesh router as the shortest path from the gateways. The routing paths are $0 \rightarrow 2, 0 \rightarrow 2 \rightarrow 5, 1 \rightarrow 3$ and $1 \rightarrow 4$.

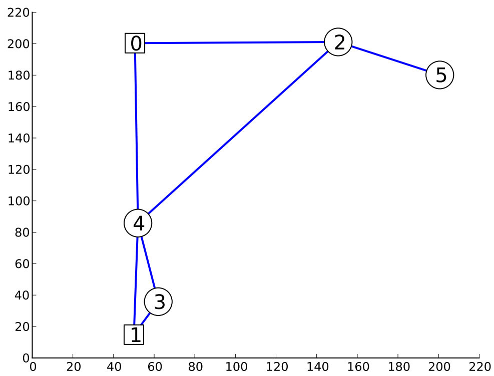

Figure 8: a network example

The transmitting power is set to 10 mW for all nodes. Each mesh router has a traffic requirement of 100 Mbp. The optimal link scheduling with rate adaptation is shown in Table 4. Each row specifies a compatible set along with the optimized MCSs, and the corresponding number of assigned time slots. The optimal number of required time slots is 16. We can also see that a link can use different MCSs in over the time slots.

Table 4: Optimal link scheduling with rate adaptation.

#	compatible sets {link, rate}	$t_{\#}$
1	{(0,2), 18}	10s
2	{(1,4), 36}	2s
3	{(2,5), 48} {(1,4), 18}	2s
4	{(0,2), 12} {(1,3), 54}	2s

Next, we consider the problem of joint optimization of link scheduling, rate adaptation, and power control. We set the maximum power to $P^{max} = 10$ mW, which is the fixed power value used for the results in the previous table. The optimal solution is given in Table 5. For each compatible set, a link belonging to the set is associated with an MCS. A node is associated with its optimized transmitting power. We can see that the transmitting power of a node varies over compatible sets. The total number of required time slots is 14, which is smaller than the case without power control. The power consumption is also greatly reduced.

Table 5: Optimal link scheduling, rate adaptation and power control.

#	compatible sets {link, rate, power}	$t_{\#}$
1	{(2,5), 18, 0.8} {(1,4), 36, 10}	2s
2	{(2,5), 48, 7.3} {(1,4), 18, 4.9}	1s
3	{(0,2), 18, 8.0} {(1,3), 54, 1.0}	11s

When routing is taken into account, optimization is conducted for link scheduling, rate adaptation, and routing together. The solution is given in in Table 6. The total number of required slots is 15, which is one less than the case without optimizing routing. In comparison to the solution without routing optimization in Table 4, the list of used compatible sets and the assigned time slots become different. One can see that an optimized routing path is $1 \rightarrow 3 \rightarrow 4$, which is not a shortest path.

Table 6: Optimal link scheduling, rate adaptation and routing.

#	compatible sets {link, rate}	$t_{\#}$
1	{(0,2), 18}	8s
2	{(0,2), 12} {(3,4), 24}	1s
3	{(2,5), 48} {(3,4), 36}	2s
4	{(0,2), 12} {(1,3), 54}	4s

Next, omni-directional antennas are replaced directional antennas at all nodes. The antenna pattern is assumed to be the one in Figure 4, in which the peak gain is 5 and the beam width is $120°$. Table 7 shows the optimal solution for joint optimization of link scheduling, rate adaptation, and routing with directional antennas. The total required time slots is brought down to 9, which is significantly better than the case with omni-directional antennas.

Table 7: Optimal link scheduling, rate adaptation and routing with directional antennas.

#	compatible sets {link ,rate, power}	$t_{\#}$
1	{(0,2), 36}	5s
2	{(0,5), 18} {(1,3), 54}	2s
3	{(1,4), 54} {(2,5), 54}	2s

Channel assignment is considered based on the compatible sets in the optimal solution of Table 6 for joint link scheduling, rate adaptation, and routing. The maximum number of interfaces in each node as well as the number of orthogonal channels are assumed to be two. The multi-channel frame derived from dynamic channel assignment is given in Table 8, and the corresponding result for static channel assignment is displayed in Table 9. One can observe that with dynamic channel assignment, only two time slots are requited, whereas four time slots are requited with static channel assignment.

Table 8: Dynamic channel assignment

Slot 1		Slot 2	
compatible sets {link,rate}	channel	compatible sets {link, rate}	channel
{(0, 2), 18}	1	{(0, 2), 12} {(3, 4), 24}	1
{(2, 5), 48} {(3, 4), 36}	2	{(0, 2), 12} {(1, 3), 54}	2

Table 9: static channel assignment

Slot 1		Slot 2	
compatible sets {link,rate}	channel	compatible sets {link, rate}	channel
{(0, 2), 18}	1	{(0, 2), 12} {(3, 4), 24}	1
Slot 3		Slot 4	
{(2, 5), 48} {(3, 4), 36}	1	{(0, 2), 12} {(1, 3), 54}	1

Next, we illustrate the max-min flow for given time $T = 1$s, with link scheduling and rate adaptation as the underlying optimization options. Algorithm 1 is used to solve the model and obtain the optimal solution, which is shown in Table 10. The max-min flow allocation vector is $f = (6.43, 6.43, 6.43, 6.43)$. In fact, one finds that the optimal compatible sets and the corresponding time proportions are the same as the solution in Table 4, by changing assigned time slots to time proportions. The reason is that the solution in Table 4 is obtained under uniform demands for all mesh routers.

Table 10: Optimal solution for max-min flow.

#	compatible sets {link, rate, power}	$t_{\#}$
1	{(0, 2), 18}	63.5%
2	{(1, 4), 36}	11.2%
3	{(1, 4), 18} {(2, 5), 48}	13.4%
4	{(0, 2), 12} {(1, 3), 54}	11.9%

Finally, the optimal solution from the model of joint link metric selection and routing in (22) is considered. The capacity of each link is given by a fixed rate QPSK 1/2, i.e., c_e =12 Mbps. The data rate requirement for each mesh router is 6 Mbps. The set of nodes and available links are shown in Figure 8. With model LMR (22), the optimal link metrics are derived; the metric of link (0,2) is two and others are one. The resulting optimal routing paths in form of ({(link),the amount of traffic}) are given in Table 11.

Table 11: Optimal solution for joint link metric selection and routing.

<table>
<tr><th>node</th><th>routing path</th><th>node</th><th>routing path</th></tr>
<tr><td>2</td><td colspan="3">{(0, 2), 3}; {(0, 4), 3},{(4, 2), 3}</td></tr>
<tr><td>3</td><td>{(1, 3), 6}</td><td>4</td><td>{(1, 4), 6}</td></tr>
<tr><td>5</td><td colspan="3">{(0, 2),3},{(2, 5),3}; {(0, 4),3},{(4, 2),3},{(2, 5),3}</td></tr>
</table>

7 Concluding remarks

In this report, a wide range of mathematical programming models for resource allocation, fair flow optimization, and metric-based routing design are presented for wireless mesh networks. For TDMA-based wireless mesh networks, models for optimal resource allocation for minimizing the length of TDMA frame are first presented. Then, the study is extended from a traffic engineering objective to address max-min fairness. Finally, joint routing and link metric optimization have been studied.

For the study of resource allocation, the derived mixed integer linear programming formulations utilize the concept of compatible set, which represents a feasible set of simultaneously active links. A sequence approach has been undertaken in the study. First, we provide the model for optimizing link transmission scheduling for optimal resource reuse in order to maximize capacity. Then, we extend the model successively to accommodate routing, power control, rate adaptation, and directional antennas. These models are non-compact due to the exponential number of compatible sets. The linear relaxation can be solved by column generation. The formulation for generating compatible sets and also two enhanced formulations are provided. Finally, we consider static as well as dynamic channel assignment and present mixed integer programming models for assigning channels to compatible sets.

From the traffic engineering viewpoint, the report addresses fairness in wireless mesh networks, in terms of max-min fairness among all flows from gateways to mesh routers. The max-min problem is formulated as a lexicographically maximization model, and an algorithm is given for model solution. The presented model provides a framework for max-min flow fairness, where optimization models for resources allocation can be embedded.

As the last part of the study, the report has studied the optimal design of routing with link metric selection. The average packet delay has been taken as the objective, and mixed integer linear programming models for choosing optimal paths for the mesh routers as well as the link metrics are developed. OSPF with ECMP, which is a common protocol feature, has been taken into account, and the piecewise linear approximation technique is used to linearize the delay function.

To conclude, the report has delivered a comprehensive study of optimization formulation for resource allocation and routing in wireless mesh networks. The models characterize the performance and capacity region within which a wireless mesh network can operate. Solving the models enable to gain fundamental understanding of the capacity aspect, leading to guidelines for engineering design of efficient and high-capacity multi-radio multi-channel wireless mesh networks.

From a research point of view, emerging lines of study are the use of interference cancellation techniques and cooperative transmission schemes for enhancing the performance of wireless systems including wireless mesh networks. These lines represent interesting topics for research and development efforts.

Glossary of Terms

WMNs	wireless mesh networks
TDMA	time division multiple access
CSMA	carry sense multiple access
MCS	modulation and coding schemes
SINR	signal to interference plus noise ratio
B&P	branch and price
B&B	branch and bound
MIP	mixed integer programming
LS	link scheduling
CSG	compatible set generation
LSR	link scheduling and routing
LSRP	link scheduling and routing with path generation
CSG/RA	compatible set generation with rate adaptation
SRA	static rate assignment
CSG/PC	compatible set generation with power control
CSG/DA	compatible set generation with directional antenna
DCA	dynamic channel assignment
SCA	static channel assignment
MMF	max-min fairness
OFDM	orthogonal frequency division multiplexing
OSPF	open shortest path first
ECMP	equal cost multi-path split rule
LMR	link metric based routing design

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